Toward AI for Lean, via metaprogramming

Robert Y. Lewis

Vrije Universiteit Amsterdam

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Introduction

What this talk is not about:

- novel AI techniques
- novel AI applications
- finished work

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What this talk is about:

- "easy" AI applications in ITP
- stress-testing Lean's tactic framework
- components of something larger

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Background: Lean

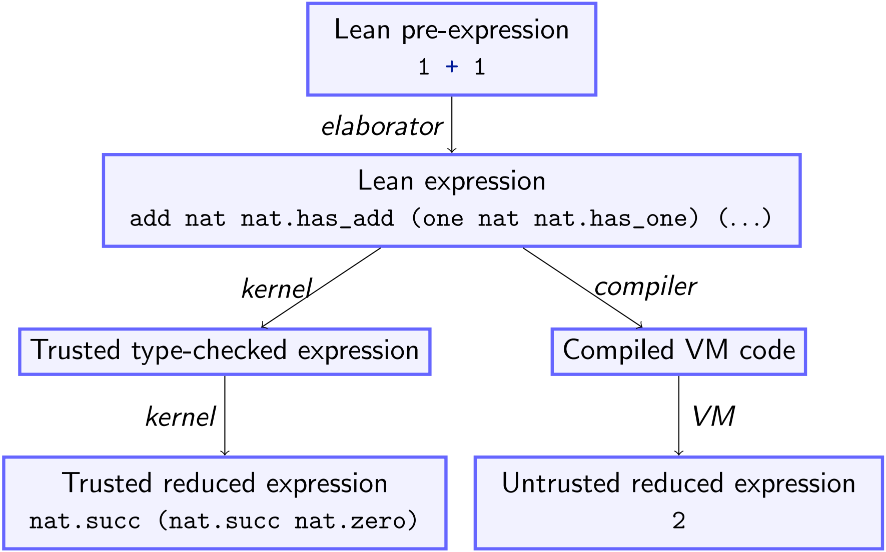
Lean is a new interactive theorem prover, developed principally by Leonardo de Moura at Microsoft Research, Redmond.

Calculus of inductive constructions with:

- non-cumulative hierarchy of universes
- impredicative `Prop`
- quotient types and propositional extensionality
- axiom of choice available

See `http://leanprover.github.io`

Expression evaluation



The Lean VM

- The VM can evaluate anything in the Lean library, as long as it is not `noncomputable`.
- It substitutes native nats, ints, arrays.
- It has a profiler and debugger.
- The VM is ideal for non-trusted execution of code.

Lean as a Programming Language

Definitions tagged with `meta` are "VM only," and allow unchecked recursive calls.

```
meta def f : ℕ → ℕ
| n := if n=1 then 1
       else if n%2=0 then f (n/2)
       else f (3*n + 1)

#eval (list.iota 1000).map f
```

Metaprogramming in Lean

Question: How can one go about writing tactics and automation?

Lean's answer: go meta, and use Lean itself.

Advantages:

- Users don't have to learn a new programming language.
- The entire library is available.
- Users can use the same infrastructure (debugger, profiler, etc.).
- Users develop metaprograms in the same interactive environment.
- Theories and supporting automation can be developed side-by-side.

Metaprogramming in Lean

The strategy: expose internal data structures as `meta` declarations, and insert these internal structures during evaluation.

```
meta constant expr : Type
meta constant environment : Type
meta constant tactic_state : Type
meta constant to_expr : expr → tactic expr
```

Tactic proofs

```
meta def p_not_p : list expr → list expr → tactic unit
| []          Hs := failed
| (H1 :: Rs) Hs :=
  do t ← infer_type H1,
     (do a   ← match_not t,
         H2  ← find_same_type a Hs,
         tgt ← target,
         pr  ← mk_app ‘absurd [tgt, H2, H1],
         exact pr)
     <|> p_not_p Rs Hs

meta def contradiction : tactic unit :=
do ctx ← local_context,
   p_not_p ctx ctx

lemma simple (p q : Prop) (h₁ : p) (h₂ : ¬p) : q :=
by contradiction
```

Applications of metaprogramming

How far can we push this framework?

- simplification and normalization
- decision procedures
- connections to external software
- superposition prover

Target: a sledgehammer for Lean, without touching the Lean source code.

Relevance filtering

Given `P : Prop`, produce a list of declarations likely to be used to prove `P`.

We need to:

- map over the environment
- build a database of declarations
- define a relevance function
- find the top k matches

```
meta def k_relevant_facts (target : expr) (k : ℕ) :
  tactic (list name) := ...
```

Relevance filtering

Lean provides:

- `meta constant get_env : tactic environment`
- `meta constant environment.fold : environment` $\to$ α $\to$ `(declaration` $\to$ α $\to$ α`)` $\to$ α
- `meta constant rb_map : Type` $\to$ `Type` $\to$ `Type`
- `meta def array : Type` $\to$ $\mathbb{N}$ $\to$ `Type`
- a VM implementation of arrays with destructive updates

For convenience, we add:

- `meta constant float : Type` and associated operations

Relevance filtering

Following Czajka and Kaliszyk (2018) we implement k nearest neighbors and sparse naive Bayes classifiers.

```
meta def feature_distance (f1 f2 : name_set) : float :=
let common := f1.inter f2 in
(common.map compute_feature_weight).sum

meta def nearest_k (features : name_set) ...
      {n} (names : array n name) (k : ℕ) :
  list (name × float) :=
let arr_prs : array n (name × float) := ⟨λ i, ...⟩,
    sorted := partial_sort
      (λ n1 n2 : name × float, float.lt n2.2 n1.2)
       arr_prs k in
if h : k ≤ n then (sorted.take k h).to_list else
    sorted.to_list
```

Relevance filtering

Downsides:

- inefficient (4-5 sec to build data structures)
- underdeveloped libraries
- depends on "hacked" version of Lean

Upsides:

- portable
- integrates with Lean library
- could potentially verify parts of the process

Computer algebra

Computer algebra systems offer many things that proof assistants lack:

- fast computation
- huge(r) libraries of functions
- ease of use
- visualization and display

Connecting Lean and Mathematica

We[1] provide:

- an extensible procedure to interpret Lean in Mathematica
- an extensible procedure to interpret Mathematica in Lean
- a link allowing Lean to evaluate arbitrary Mathematica commands, and receive the results
- tactics for certifying results of particular Mathematica computations
- a link allowing Mathematica to execute Lean tactics and receive the results

[1] RL + Minchao Wu (Australia National University)

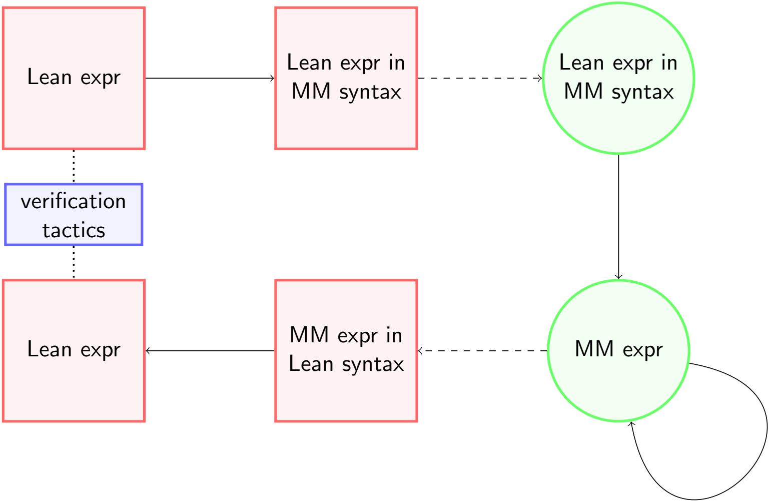
Connecting Lean and Mathematica

The idea: many declarations in Lean correspond *roughly* to declarations in Mathematica.

We can do an approximate translation back and forth and verify post hoc that the result is as expected.

Correspondences, translation rules, checking procedures are part of a mathematical theory.

Calling Mathematica from Lean

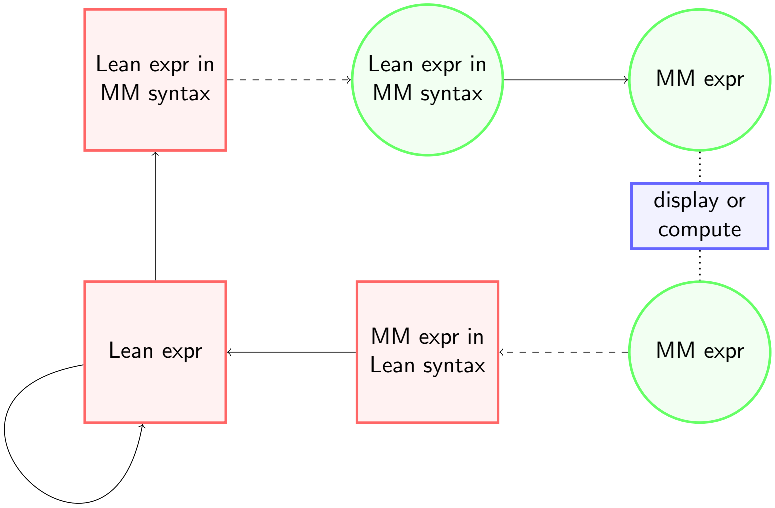


A relevance filter via Mathematica

Mathematica has various tools for “black box” machine learning.

We can build the data structures in Lean and send them to Mathematica for processing.

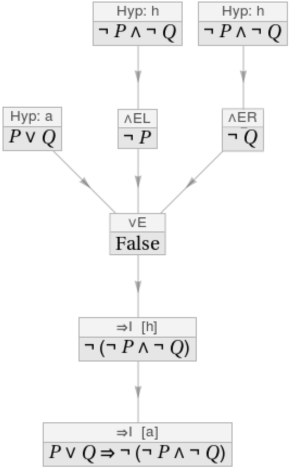
Calling Lean from Mathematica



Calling Lean from Mathematica

We can add rules to (try to) translate the resulting proof.

```
DiagramOfFormula[
 ForAll[{P, Q},
  Implies[
   Or[P, Q],
   Not[And[Not[P], Not[Q]]]
  ]
 ]
]
```



Calling Lean from Mathematica

With this link, we can access either relevance filter from within Mathematica.

- State a theorem in Mathematica, translate to Lean, and get relevant facts.
- Idea: make CAS useful for proof exploration instead of just computational exploration.

Calling Lean from Mathematica

```
SelectLeanPremises[ForAllTyped[{x, y}, real, Implies[x < y, Exists[{z}, And[x < z, z < y]]]]]

ennreal.densely_ordered._match_1 : ∀ (r : ℝ), 0 ≤ r → ∀ (p : ℝ), 0 ≤ p →(∃ (a : ℝ),
  r < a ∧ a < p) → (∃ (a : ennreal), a < ennreal.of_real p ∧ ennreal.of_real r < a)
exists_pos_of_rat : ∀ {r : ℝ}, 0 < r → (∃ (q : ℚ), 0 < q ∧ of_rat q < r
```

Thanks for listening!

Questions, then skiing.