

80-110: The Nature of Mathematical Reasoning

Spring 2015

Rob Lewis

1 Basic Info

- Location: Doherty Hall 1115
- Instructor: Rob Lewis (rlewis1@andrew.cmu.edu)
- Office: Doherty Hall 4302D
- Mailbox: Baker Hall 135
- Office hours: MWF 10:30-11:30 or by appointment
- Course website: <http://www.andrew.cmu.edu/user/rlewis1/80110/>

2 Course Description

This course is intended to serve as an introduction to what we could broadly call “metamathematics.” Without delving too deeply into any particular mathematical theory (like calculus or topology), we’ll learn some interesting facts about mathematical systems in general, and see how these ideas can be used in real-life problems. To do this, we will need to understand what exactly mathematics “is,” so to speak. To this end, the course will involve both some math and some philosophy (although no background in either is required).

A central point of this class is the idea of mathematical rigor. Mathematics is, ideally, something that is perfectly precise; there is no room for ambiguity, and every question in the “language” of math has a correct answer. We’ll talk about how mathematics can be used to add precision to situations that lack it, to clarify questions and make them answerable. At the same time, one can ask questions *about* mathematics that seem to be very open-ended. We’ll examine what makes a question essentially mathematical, and what makes a question philosophical.

The course will be broadly organized in four parts. First, we’ll discuss some historical background and motivation for the topic: we’ll look at some mathematical and philosophical work from the ancient Greeks, contributions they made, and problems they ran into. We’ll then look more generally at the notion of a mathematical “theory,” informally and formally. What are axioms? Definitions? Theorems? This will lead us into a discussion of propositional and first-order logic. With this framework, we’ll be able to discuss some well-known fallacies and paradoxes, and see how the machinery of logic (and mathematics

in general) allows us to resolve these. If there is time, we'll finish the course by talking about set theory, a common foundation for mathematics.

If there are any (even only slightly related!) topics that you think would be interesting to discuss, please let me know! Our schedule is flexible and I'm open to suggestions.

3 Assignments

There will be readings and/or problems assigned daily. Readings will be short and serve as background for the next day's lecture; I will distribute reading schedules for each week in advance. "Problem sets" will consist of one to three problems, when we're in a part of the course where this is applicable, and will generally be due a few classes after they are assigned. Be prepared to share your solutions with the class, and to discuss any problems that you found difficult.

There will be two quizzes and one exam. The first quiz will cover roughly the first and second parts of class, on history and general definitions. The second quiz will test the applications of these ideas to fallacies and paradoxes. The exam will cover the more technical ideas about formal logic. Note that the exam occurs at the middle of the course, between the two quizzes!

The quizzes will be an hour long, on your own time. The exam will be one class period. Broadly speaking, the quizzes will be more "philosophical" (explaining ideas and terms), while the exam will be more "mathematical."

Finally, there will be an optional "research" paper, at least 1500 words, due at the end of the semester. I will distribute possible topics partway through the course.

4 Attendance and Participation

This will be a small class. Attendance and participation are very important. I expect that everyone will stay fully up to date on the readings and problem sets, and will contribute to class discussions. As you are reading, try to form an opinion on the topic: how does it compare to earlier readings? Do you agree with the author's argument (if there is one)? Can you think of examples of the topic, or counterexamples? Thinking about these things in advance will help move along class discussions.

I do not have a strict attendance policy, but I expect you to be in class every day. If you will be absent, please let me know by email as far in advance as possible. An occasional absence is fine, but a regular habit of missing class is not and will affect your participation grade.

5 Grading

Homework problems will be graded out of four points, corresponding to the following scale. Precise arguments are essential!

Since the problem sets are short and the class moves quickly, late submissions are strongly discouraged. You are allowed a total of three "late days" over the course of the semester. This means that either one assignment can be turned in three days late, or three assignments can be turned in one day late each, etc,

without question or penalty. After this, late assignments will not be accepted. Quizzes must be turned in on the day they are due, no exceptions or late days allowed.

Homeworks will be graded on the following scale:

- 0: not attempted
- 1: little work done/little understanding shown
- 2: some progress, but significant mistakes
- 3: mostly correct argument, but small mistakes or vague wording
- 4: correct argument, clearly worded

Final grades will be determined by quiz grades (15% each), exam grade (30%), homework grades (30%), and participation (10%). The optional final paper is worth 10%.

6 Schedule

Since this is a small class, the schedule is malleable and subject to change. I will make sure to provide assignments and important dates in advance. Right now, there are a few dates to keep in mind:

- Monday, 1/12: first day of class
- Monday, 1/19: MLK day, no class
- Wednesday, 2/20: First quiz (tentative)
- 3/6-3/13: Spring break
- Friday, 4/3: Exam (tentative)
- Friday, 4/17: Spring carnival, no class
- Friday, 5/1: Last day of class, second quiz (tentative)
- Friday, 5/13: Optional papers due

7 Classroom Policies

Laptops and tablets are allowed in class, but it should go without saying that they should be used for class purposes only. Please keep your phones put away.

You are encouraged to work together on homework assignments, but everything you write up and turn in must be done on your own. A good policy to follow is the “erased blackboard” policy: solve problems together, but write up your solutions alone without visual reference to your communal work. Please make a note on your assignments of who you worked with, if anyone.

Quizzes, exams, and papers must represent your work only. No collaboration is allowed. Plagiarism and cheating are serious academic offenses with serious consequences. If you are discovered engaging in either behavior, you will receive a failing grade on the assignment or exam in question, and further disciplinary action will be taken, in accord with the university’s policies.