

The Lean 3 Mathematical Library (mathlib)

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Lean

- ▶ An open source interactive theorem prover developed primarily by Leonardo de Moura (Microsoft Research)
- ▶ Focuses on software verification and **formalized mathematics**
- ▶ Based on Dependent Type Theory
 - ▶ Classical, non-HoTT
- ▶ Lean 3 includes a powerful metaprogramming infrastructure for Lean in Lean

A Short History of Lean and mathlib

Several major versions:

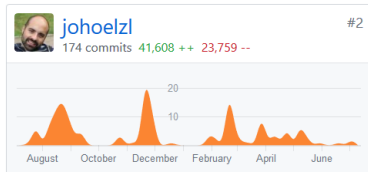
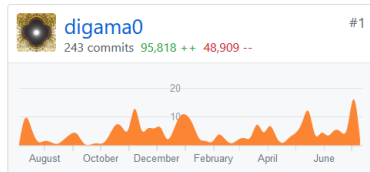
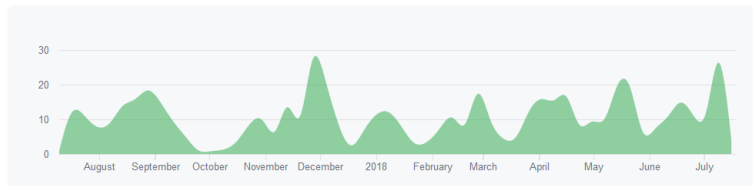
- ▶ Lean 1 (no public release)
- ▶ Lean 2 (2015) – includes HoTT mode
- ▶ Lean 3 (2017)
- ▶ Lean 4 in development

- ▶ The Lean 2 math library was developed by Jeremy Avigad, Floris van Doorn et al.
- ▶ Lean 3 is not backwards compatible with Lean 2, and the decision was made to start again taking advantage of significant new features
- ▶ `mathlib` is the latest version of the Lean 3 math library, developed primarily by Mario Carneiro and Johannes Hölzl

Jul 16, 2017 – Jul 25, 2018

Contributions: **Commits** ▾

Contributions to master, excluding merge commits



Mathlib goals

Two main goals:

- ▶ (CS) To build a standard library for lean as a programming language
 - ▶ To support verified programming and proven-correct algorithms
 - ▶ To support and provide tactics and decision procedures for proof automation
- ▶ (Math) To build a library of formalized mathematics, and support users doing the same

These goals complement each other, it is not just two libraries in one

Mathlib vs the core library

- ▶ Lean itself has a library, which is even more geared towards CS applications and MS users
- ▶ Mathlib is developed on top of this library, and is fully compatible with it, but significantly expands on the mathematics, the (Lean) programming, and the tactics
- ▶ The core lean library is currently frozen while Lean 4 is under development, but mathlib is very active

What is in mathlib?

Lean mathematical components library

584 commits

3 branches

0 releases

18 contributors

Apache-2.0

Branch: master ▾




















New pull request

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 cipher1024 and johoezl feat(category/traversable): basic classes for traversable collections	Latest commit f9cf9d3 16 days ago
 algebra	feat(algebra/pi_instances): more pi instances a day ago
 analysis	refactor(analysis/enreal): split and move to data.real 4 days ago
 category	feat(category/traversable): basic classes for traversable collections 19 hours ago
 computability	fix(computability/turing_machine): missed a spot 4 days ago
 data	refactor(data/nat/gcd): simplify proof of pow_dvd_pow_iff 3 days ago
 docs	doc(wip): finite map (#215) [ci-skip] a day ago
 group_theory	fix(group_theory/group_action): move is_group_action out of namespace 6 days ago
 linear_algebra	refactor(data/polynomial): move polynomials to data; replace monomial... 7 days ago
 logic	refactor(data/set/countable): define countable in terms of encodable 8 days ago
 meta	feat(tactic): add `wlog` (without loss of generality), `tauto`, `auto... 5 months ago
 number_theory	refactor(data/set/finite): use hypotheses for fintype assumptions 2 months ago
 order	feat(algebra/pi_instances): more pi instances a day ago
 pending	feat(data/real): reals from first principles 6 months ago
 ring_theory	fix(*): fix build 7 days ago
 set_theory	refactor(data/set/basic): rename set.set_eq_def -> set.ext_iff 8 days ago
 tactic	chore(tactic/interactive): change swap so it does what it says 4 days ago
 tests	feat(tactic/h_generalize): remove `cast` expressions from goal (#198) 5 days ago
 .aitianore	refactor(*): import content from lean/librарv/data and librарv dev a year ago

Datatypes: data

These are all computable where applicable, and so can be used in programming contexts

- ▶ $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ – classical number types
- ▶ `list α` – finite sequences on α , a.k.a linked lists
- ▶ `multiset α` – lists up to permutation (quotient type)
- ▶ `finset α` – multisets with no duplicates, used to define finiteness of types and sets
- ▶ `array n α` – arrays of fixed length n (implemented efficiently in C++)
- ▶ `vector α n` – lists of fixed length n (proven isomorphic to `array n α`)

Datatypes: data

- ▶ $\alpha \simeq \beta$ (equiv) – equivalence/isomorphism of types
- ▶ $\alpha \hookrightarrow \beta$ (embedding) – injective functions
- ▶ `encodable` α – a map from α to \mathbb{N} with partial inverse (Gödel numbering)
- ▶ `stream` α , `seq` α , `wseq` α – different kinds of coinductive lists
- ▶ `pos_num`, `num`, `znum` – binary natural numbers (for kernel computation)
- ▶ `option` α – optional values, nullable types
- ▶ `roption` α , `pfun` $\alpha \beta$ – partial values, functions with precondition (noncomputably isomorphic to `option` α)

Algebraic typeclasses: algebra

- ▶ semigroup, monoid, group, semiring, ring, domain, euclidean_domain, field – algebraic structures
- ▶ add_zero : $\forall x, x + 0 = x$ – theorems in the first order theory of these structures
- ▶ instances showing that the product of groups is a group, the product of rings is a ring, etc

Order structures, sets: `order`, `data.set`

- ▶ `preorder`, `partial_order`, `linear_order`, `lattice`, `bounded_lattice`, `complete_lattice`, `conditionally_complete_lattice` – order structures
- ▶ `set α` – the collection of all subsets of α , encoded as functions $\alpha \rightarrow \mathbf{Prop}$
- ▶ `filter α` – the collection of all filters on α (which is a complete lattice and a monad)

Topology: analysis.topology

- ▶ `topological_space α` – a type equipped with an `is_open` predicate
- ▶ `nhds a` – the neighborhoods filter
- ▶ `map`, `induced`, `coinduced` – topological constructions
- ▶ `closed`, `compact`, `continuous` – topological definitions
- ▶ `t1_space`, `t2_space`, `regular_space`, `separable_space`, `first/second_countable_topology` – topological properties
- ▶ `topological_add_group`, `topological_semiring` – topological algebraic structures

```
instance [t1 : top  $\alpha$ ] [t2 : top  $\beta$ ] : top ( $\alpha \times \beta$ ) :=  
induced prod.fst t1  $\sqcup$  induced prod.snd t2
```

Theories

- ▶ `uniform_space`, `metric_space`
- ▶ `measure_theory` – measure spaces, measurable functions, outer measures, measures, Lebesgue measure
- ▶ `group_theory` – group actions, subgroups, quotient groups
- ▶ `ring_theory` – ideals and local rings
 - ▶ much more work is currently happening in the community but not yet in `mathlib`; Kevin Buzzard is working on schemes and perfectoid spaces
- ▶ `computability` – primitive and partial recursive functions, Turing machines, universality and the halting problem
- ▶ `number_theory` – the Pell equation, Diophantine equations
- ▶ `set_theory` – cardinal and ordinal numbers, computable ordinal notations, large cardinals, a model of ZFC

Tactics: `tactic.interactive`

Mostly minor improvements to the lean core tactic library

- ▶ `rcases`, `rintro` – cases with a pattern
- ▶ `finish` (Jeremy Avigad), `tauto` (Simon Hudon) – general purpose automation
- ▶ `norm_num` – decision procedure for numeric calculation
- ▶ `ring`, `ring2` – decision procedure for rings

There are some more tactics available in the community:

- ▶ `super` – superposition prover (Gabriel Ebner)
- ▶ `cooper` – decision procedure for Presburger arithmetic (Seul Baek)

`docs/tactics.md` has a more complete listing

Future work

- ▶ Everything!
- ▶ Need more basic analysis – derivatives and integrals
- ▶ Almost no number theory except what was needed for MDRP
- ▶ Geometry and trigonometry completely absent
- ▶ Many basic data structures are missing (binary search trees, association lists) because some techniques like memoized thunks are waiting for lean 4

Thank you!

<https://github.com/leanprover/mathlib>